

REVIEWS

Introduction to Mathematical Fluid Dynamics. By R. E. MEYER. Wiley-Interscience, 1971. 185 pp. £6.55.

Fluid Dynamics. By R. VON MISES and K. O. FRIEDRICHS. Springer-Verlag, 1971. 353 pp. \$6.50.

It is questionable whether the *Introduction to Mathematical Fluid Dynamics* by R. E. Meyer is an introduction, but it is certainly mathematical. On page 4 we read the following definition: “Let Ω_0 be any open, bounded point set in E^3 ‘occupied by the fluid’ at time $t = 0$. ‘Fluid motion’ shall mean a transformation H_t on the closure $\bar{\Omega}_0$ into E^3 such that the point set $H_t\Omega_0$ is that occupied by ‘the same fluid’ at time t .” Thus we are quickly introduced to the author’s choice of presentation and some readers will be able to decide from that one definition whether or not this book is for them.

The first half is devoted to the basic principles which govern the flow of an inviscid incompressible fluid. All the important results and theorems are there, though the treatment is highly condensed, covering in 76 pages the sort of ground to which Lamb devoted 300 pages. This is achieved by various means. An understanding of set theory and tensor calculus is assumed. Some results are stated without proof, such as “the bounding surface of a body of fluid always consists of the same fluid particles”. This is first translated into mathematical form, namely $\partial\Omega_t = H_t\partial\Omega_0$ if $\Omega_t = H_t\Omega_0$, and a reference to *Differential and Integral Calculus* by Courant given for the proof. Some results are presented as postulates. For example, postulate VI reads “A vector field \mathbf{f} and a tensor field with components p_{ij} are defined on the closure of any fluid domain Ω_t with regular boundary surface $\partial\Omega_t$ so that

$$\frac{D}{Dt} \int_{\Omega_t} \rho v_i dV = \int_{\Omega_t} \rho f_i dV + \int_{\partial\Omega_t} p_{ij} n_j dS.”$$

Some results are presented as exercises for the reader, such as the theorem of Blasius for the lift and drag on a two-dimensional cylinder. Applications are severely restricted, in fact they amount only to a discussion of some of the usual potential flow patterns and a quite thorough discussion of the line vortex and its application to the lifting line theory of the three-dimensional wing.

The second half of the book deals with the real fluid. The basic equations have now been introduced, set theory is no longer required and the presentation becomes more conventional. The incompressible viscous fluid is dealt with in 30 pages. There is a discussion of the viscous contribution to the stress tensor for a Newtonian fluid, a useful discussion, not found in many books, of the viscous boundary condition at a solid boundary from the point of view of kinetic theory, and a derivation of Poiseuille’s result for flow in a pipe. With this introduction the author proceeds to an account of matched asymptotic expansions, leading to boundary-layer theory, the Blasius result for a flat plate, wakes and jets.

Next there are 10 pages on rotating fluids, dealing with the Ekman layer and the Taylor–Proudman theorem. Finally there are 30 pages on compressible

fluids with an account of flow in a Laval nozzle, simple waves and shock waves in one-dimensional unsteady motion, and Becker's solution for the shock structure when the Prandtl number is equal to 0.75.

This is an ambitious amount of ground to cover in an alleged introduction, although the author rightly limits himself in the number of applications so that adequate space is available for the chosen topics. But most students need to spend more time studying the simpler aspects of a subject like fluid dynamics before they can cope with the difficult problems highlighted in this book. As for the set theory formalism of the basic definitions and theorems, it is true that work is proceeding in modern fluid dynamics for which such formalism is a prerequisite, rather than a luxury. Whether or not a similar statement is appropriate for an introduction to the subject depends on whether or not one agrees with the author that "For mathematics students such a treatment helps to dispel the all too common impression that the whole subject is built on a quicksand of assorted intuitions."

Fluid Dynamics, by R. Von Mises and K. O. Friedrichs, is twice the size of Meyer's book and covers approximately the same topics. The style, however, is very different and one is struck by the similarity in scope and the contrast in presentation of these two books.

We read in the preface to *Fluid Dynamics* that it is based, without modification apart from corrections, on notes prepared in mimeograph form for a course of lectures at Brown University during the summer of 1941.

The first half of the book consists of three chapters by the late Professor Von Mises on the inviscid incompressible fluid. The first chapter of 30 pages deals with the basic principles, developing the usual equations and theorems in conventional style. There follows 160 pages devoted, in effect, to the flow of an ideal fluid past two-dimensional wings (chapter 2) using conformal transformation, and three-dimensional wings (chapter 3) using lifting line theory. This is as complete an account of the subject as will be found in any student text. It is most useful as a reference section, but the demands of other aspects of fluid dynamics would prevent today's students from studying the subject in such detail.

The second half of the book, chapters 4 and 5, are by Professor K. O. Friedrichs. Chapter 4 is concerned with the incompressible viscous fluid. Not surprisingly, there are many references to Goldstein's *Modern Developments in Fluid Dynamics*. There are also Friedrichs's own inimitable contributions such as the original model equation illustrating matched asymptotic expansions, which introduces boundary-layer theory. The same model equation is used by Meyer and by other authors. The final chapter deals with compressibility and covers much the same ground as does Meyer's book. Friedrichs was a pioneer in making available to the student a connected account of this branch of fluid dynamics, and this chapter contains the seeds from which the classical text *Supersonic Flow and Shock Waves* presumably emerged.

With the qualification that the material was prepared 30 years ago, and that the content and presentation of fluid dynamics has changed in that time, one might welcome for reference purposes this collection of lectures from such eminent mathematicians.

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